MATH 323 (calc III)

Instructor: Chris E.

Email: eppolito@math.Binghamton.edu ☆ come and talk.

[ASK] Textbook: Multivariable Calculus (don't need a physical copy)

<u>Website</u>: Webassign.net. (do need this) more info later in the email
absolutely need

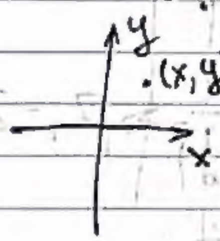
Gradescope: assignment submission (1st assignment as many times as needed to get a full mark) when ???.

Syllabus: his website (in the email)
(read) + practice problems

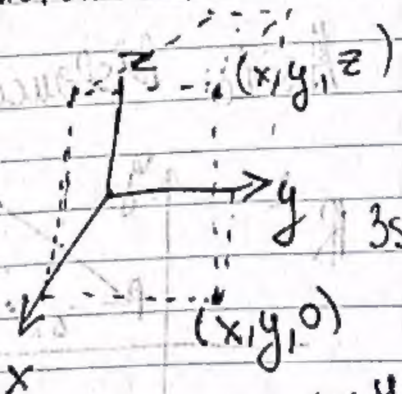
§12.1 Coordinates in 3-Space

IDEA: Extend our calc 1 and 2 to functions with several variables

Some geometry in 3-space



2-space = $\mathbb{R}^2$



(a box)

3space = $\mathbb{R}^3$

(x,y,z) - coordinates of the point $P=(x,y,z)$

I. coordinates planes
planes where a selected coordinate is 0

a coordinate plane is a set of points in which specified coordinate is 0

· Ex: The xy-plane (aka the $z=0$ plane) in $\mathbb{R}^3$ is

$\Pi = \{ P=(x,y,z) \in \mathbb{R}^3 : z=0 \}$

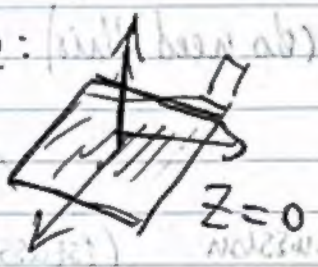
Ex: The yz-plane is

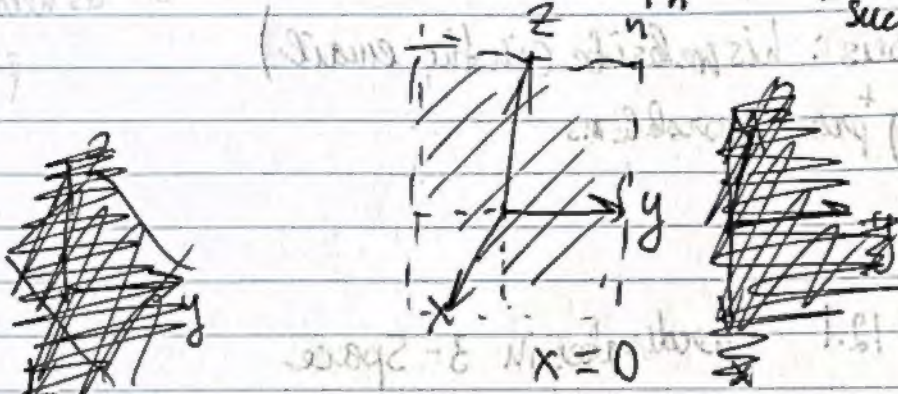
$\{ P=(x,y,z) \in \mathbb{R}^3 : x=0 \}$

$\in$ "in"

: "such that"

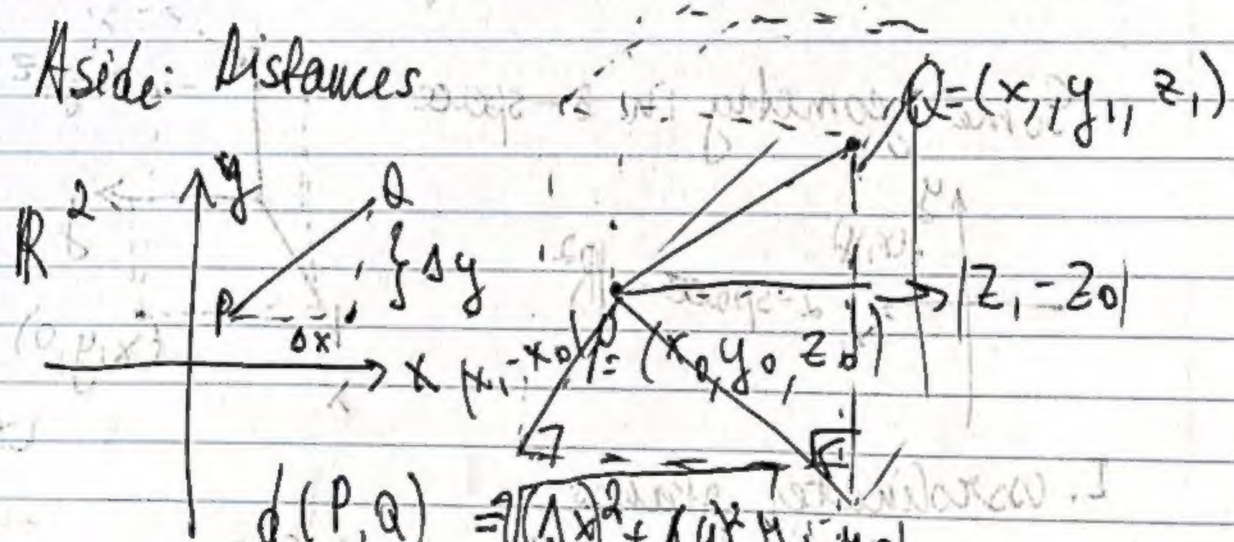
Picture:





x → out of the page

Aside: Distances



$d(P,Q) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$\sqrt{x} > 0$ if it exists

in a mespena Nugarspa we do not need to claim absolute values

$$d(P,Q) = \sqrt{(\sqrt{(x_1-x_0)^2+(y_1-y_0)^2})^2 + (z_1-z_0)^2} =$$

$$= \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

Thm (distance formula): For $P(x_0, y_0, z_0)$ and $Q(x_1, y_1, z_1)$ in 3 space the distance between P and Q is

$$d(P,Q) = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

II. Spheres

Let $r>0$ and let $P \in \mathbb{R}^3$

the sphere of radius r centered at P is $S =$
$= \{Q \in \mathbb{R}^3 : d(P,Q) = r\}$

If P has coordinates $P = (x_0, y_0, z_0)$, the

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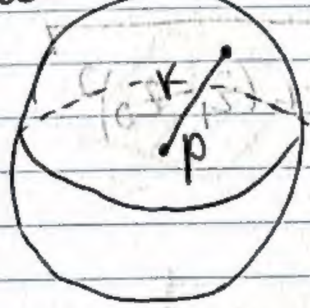
$$S = \{Q \in \mathbb{R}^3 : d(P,Q) = r\} = \{(x_1,y_1,z_1) \in \mathbb{R}^3 : \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2} = \underset{>0}{r}\} =$$

$$= \{(x_1,y_1,z_1) \in \mathbb{R}^3 : (x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2 = r^2\}$$

Picture:



NB: Spheres are "surface of a hollow ball". Not solid

A solid ball is satisfied by

$$(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2 \le r^2$$

NB means "not well"

$\{(x,y,z,w) : x,y,z,w \in \mathbb{R}\}$

Note: Everything we have done so far has analogues in higher dimensions as well. $\{(x,y,z,w) : x,y,z,w \in \mathbb{R}^4\}$

For example (e.g.), there is a 4-space $\mathbb{R}^4$, and it has a distance formula

$$d((x_0,y_0,z_0,w_0),(x_1,y_1,z_1,w_1)) =$$

$$= \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2+(w_1-w_0)^2}$$

Pictures are hard though

§ 12.2 : Vectors

Definition: a vector in $\mathbb{R}^3$ is a directed line segment, where we regard 2 linear segments as equal (or are equivalent) when they are "<u>linear shifts</u>".